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Non-ideal effects on the typical trailing edge shock pattern of ORC turbine blades

D. Vimercati[a,c], G. Gori[a,c], A. Spinelli[b,c] and A. Guardone[a,c]

Politecnico di Milano - Italy
[a] *Department of Aerospace Science and Technology*
[b] *Energy Department*
[c] *CREA Lab*

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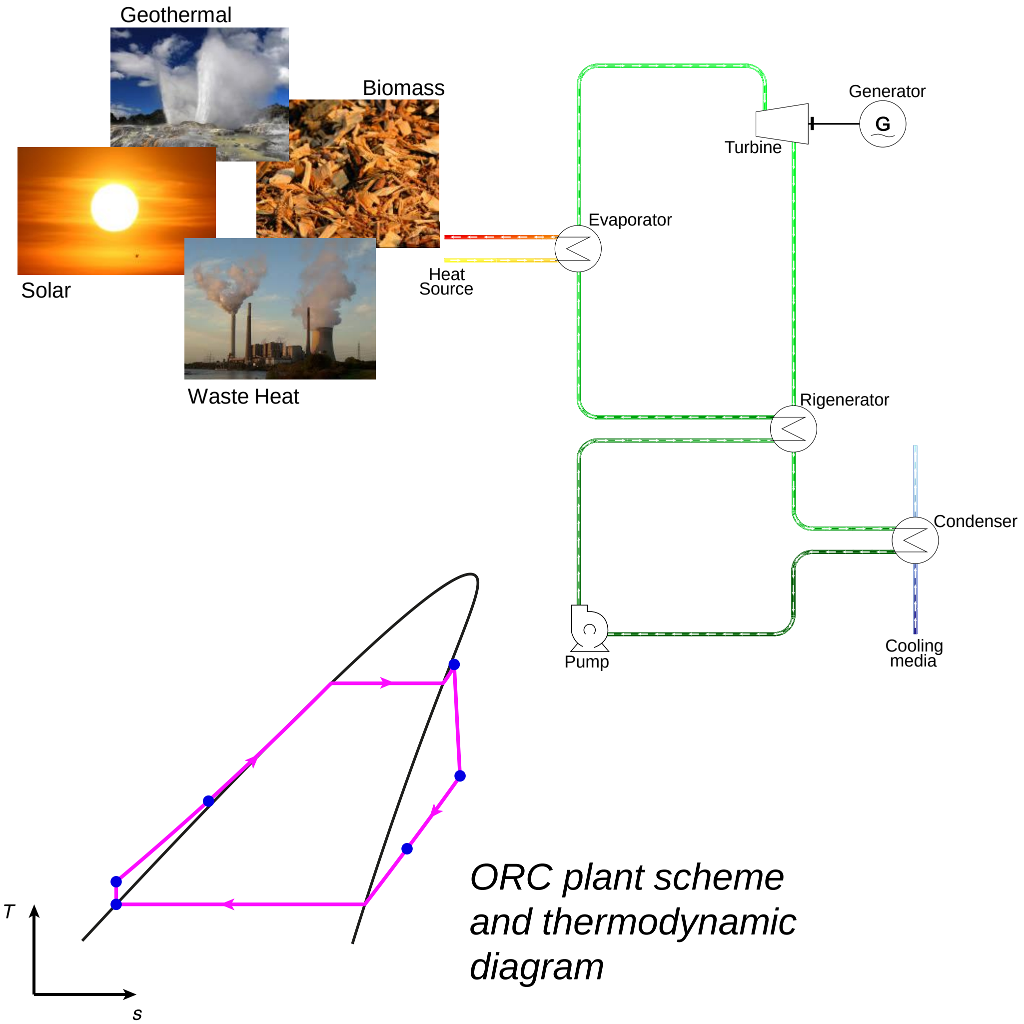
MOTIVATIONS



ORC plant scheme and thermodynamic diagram

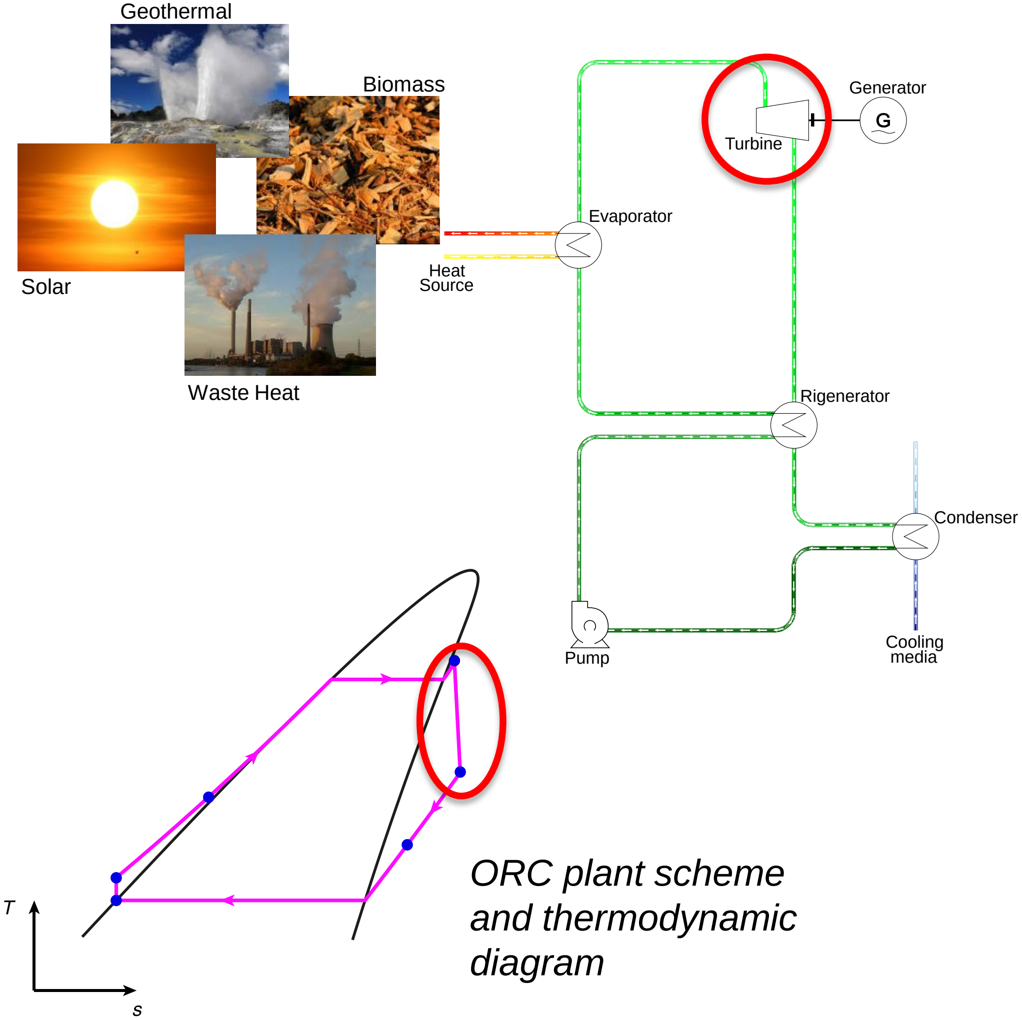
ORC attractive features

- Adaptability to various (low temperature) hea sources
- Lower complexity wrt steam cycle
- Turbine technical advantages wrt steam turbine (lower rmp, lower pressures, no erosion)
- High flexibility
- …

ORC challenges

- Choice of suitable working fluid
- Transient phenomena
- Complex thermodynamic modelling of the working fluid
- Heat exchangers and turbine design
- …

MOTIVATIONS



ORC plant scheme and thermodynamic diagram

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- ○ Adaptability to various (low temperature) hea sources
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- ○ High flexibility
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ORC challenges

- ○ Choice of suitable working fluid
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- ○ Complex thermodynamic modelling of the working fluid
- ○ Heat exchangers and turbine design
- ○ …

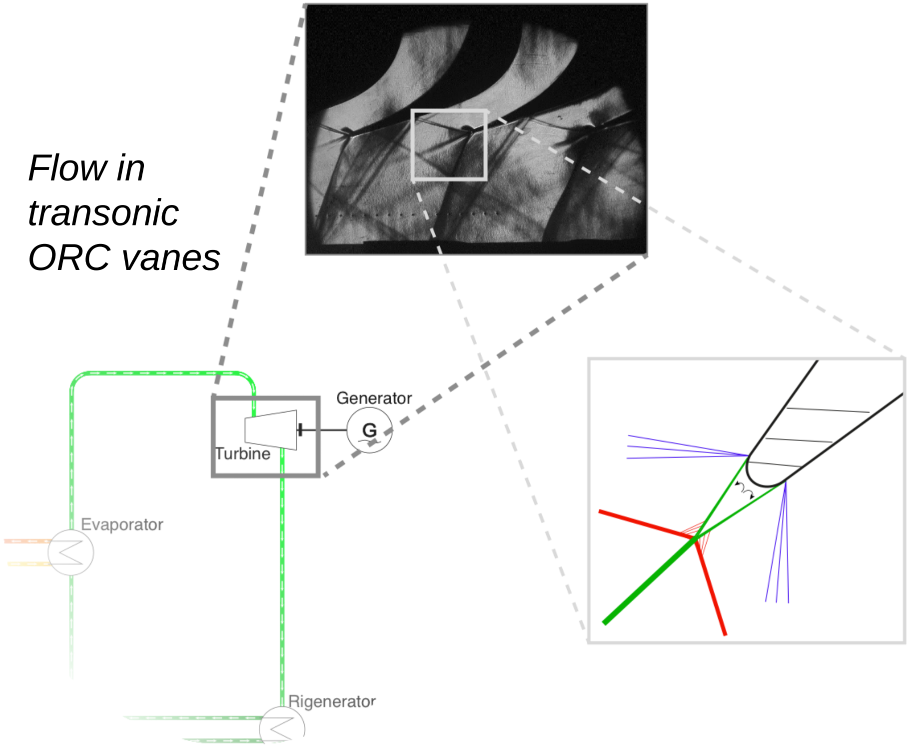
MOTIVATIONS

ORC turbine

- ○ Typically few stages (often one only)
- ○ High pressure ratio

- ○ Design expansion through the non-ideal regime: low values of the speed of sound → highly supersonic flow
- ○ Shock waves: fish-tail shocks, post-expansion, off-design
- ○ Large contribution of inviscid loss to total loss

Flow in transonic ORC vanes



Research question

How do non-ideal effects across oblique shocks impact on the design of ORC turbines?

PRESENTATION OUTLINE

→ Introduction: NICFD

→ Methodology

→ Oblique shocks in the non-ideal flow regime

→ Application: oblique shocks in siloxane MDM

→ Discussion and concluding remarks

INTRODUCTION

Non-Ideal Compressible Fluid Dynamics

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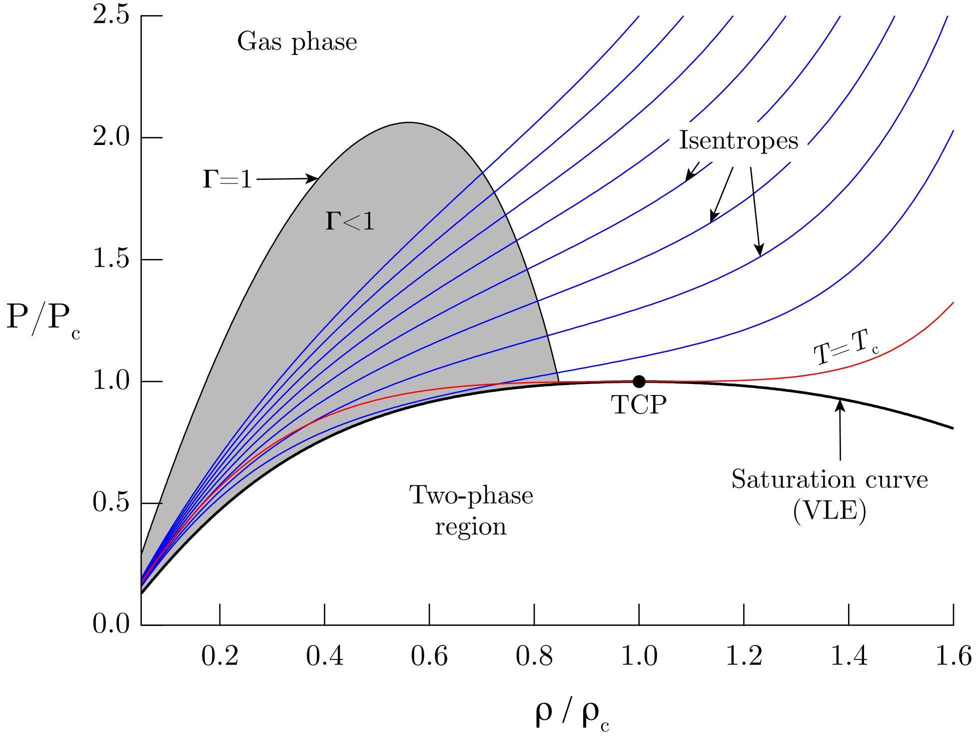
$Pv \neq RT$



Fluid: MDM (RefProp)

Features

- Subject: dense vapours, supercritical fluids, two-phase compressible flows
- Compressibility
- Phase transition
- Critical point

Application

- ORC
- Supercritical CO_2
- Refrigeration
- Oil & Gas compression/expansion
- …

INTRODUCTION

Non-Ideal Compressible Fluid Dynamics

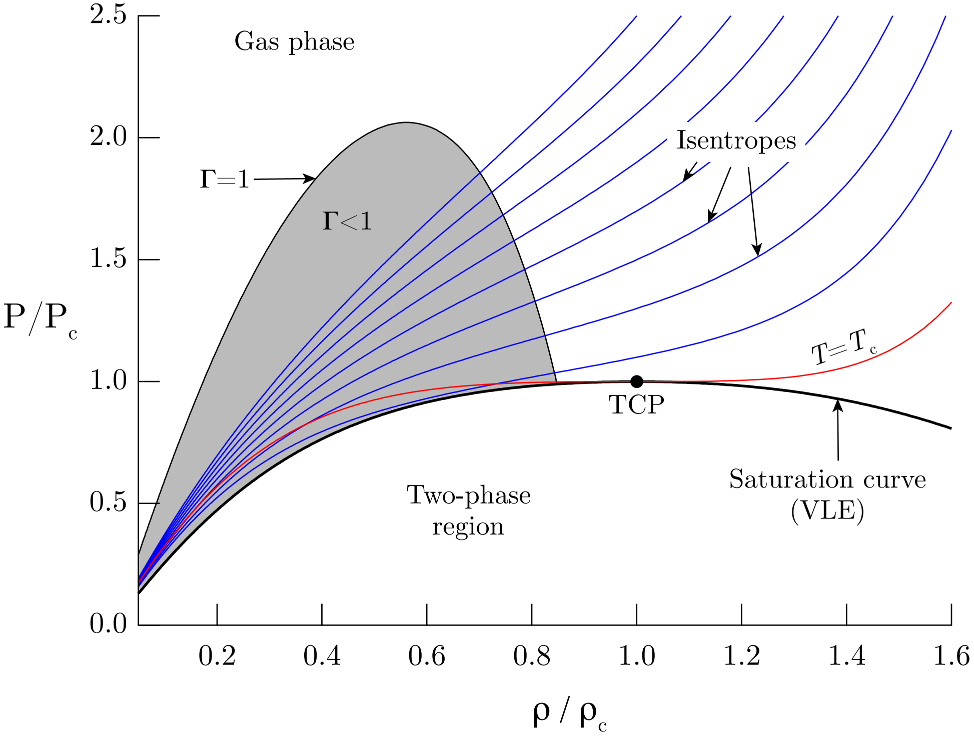
||

Pv ≠ RT

Measure of non-ideality in compressible flows:

The fundamental derivative of gasdynamics (Thompson 1971)

$$\Gamma = 1 + \frac{\rho}{c}\left(\frac{\partial c}{\partial \rho}\right)_s = 1 + \frac{\rho}{c}\left(\frac{\partial^2 P}{\partial \rho^2}\right)_s = \frac{v^3}{2c^2}\left(\frac{\partial^2 P}{\partial v^2}\right)_s$$

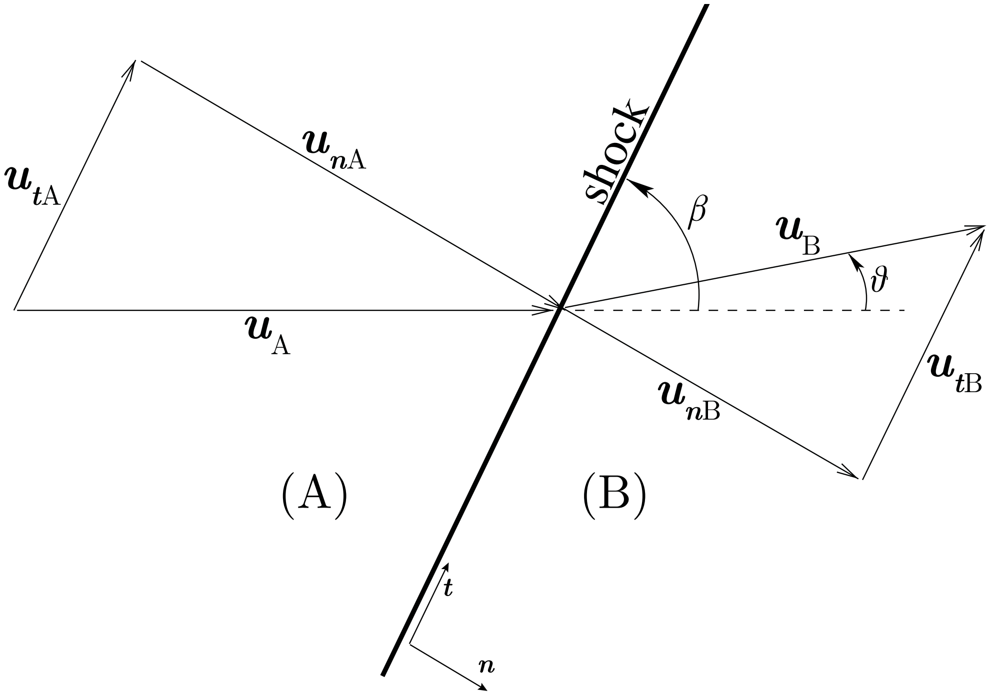


Fluid: MDM (RefProp)

Gasdynamic regimes

- $\Gamma > 1$ Ideal regime
- $\Gamma < 1$ Non-Ideal regime
  - $0 < \Gamma < 1$ Non-Ideal classical regime
  - $\Gamma < 0$ Non-Classical regime

METHODOLOGY



Rankine-Hugoniot relations

$$h_{\mathrm{A}} - \frac{1}{2}P_{\mathrm{A}}(v_{\mathrm{A}} + v_{\mathrm{B}}) = h_{\mathrm{B}} - \frac{1}{2}P_{\mathrm{B}}(v_{\mathrm{A}} + v_{\mathrm{B}})$$

$$\sqrt{-\frac{(P_{\mathrm{B}} - P_{\mathrm{A}})}{(v_{\mathrm{B}} - v_{\mathrm{A}})}} = \rho_{\mathrm{A}} |\boldsymbol{u}_{\mathrm{A}}| \sin\beta$$

$$\rho_{\mathrm{A}} \tan\beta = \rho_{\mathrm{B}} \tan(\beta - \theta)$$

$$|\boldsymbol{u}_{\mathrm{A}}| \cos\beta = |\boldsymbol{u}_{\mathrm{B}}| \cos(\beta - \theta)$$

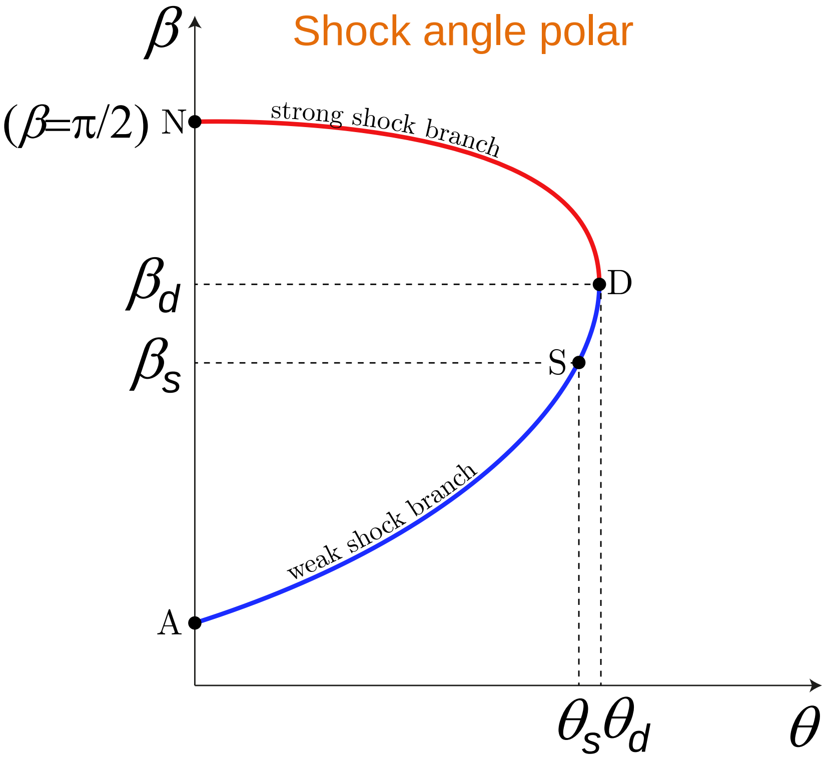
Admissibility conditions

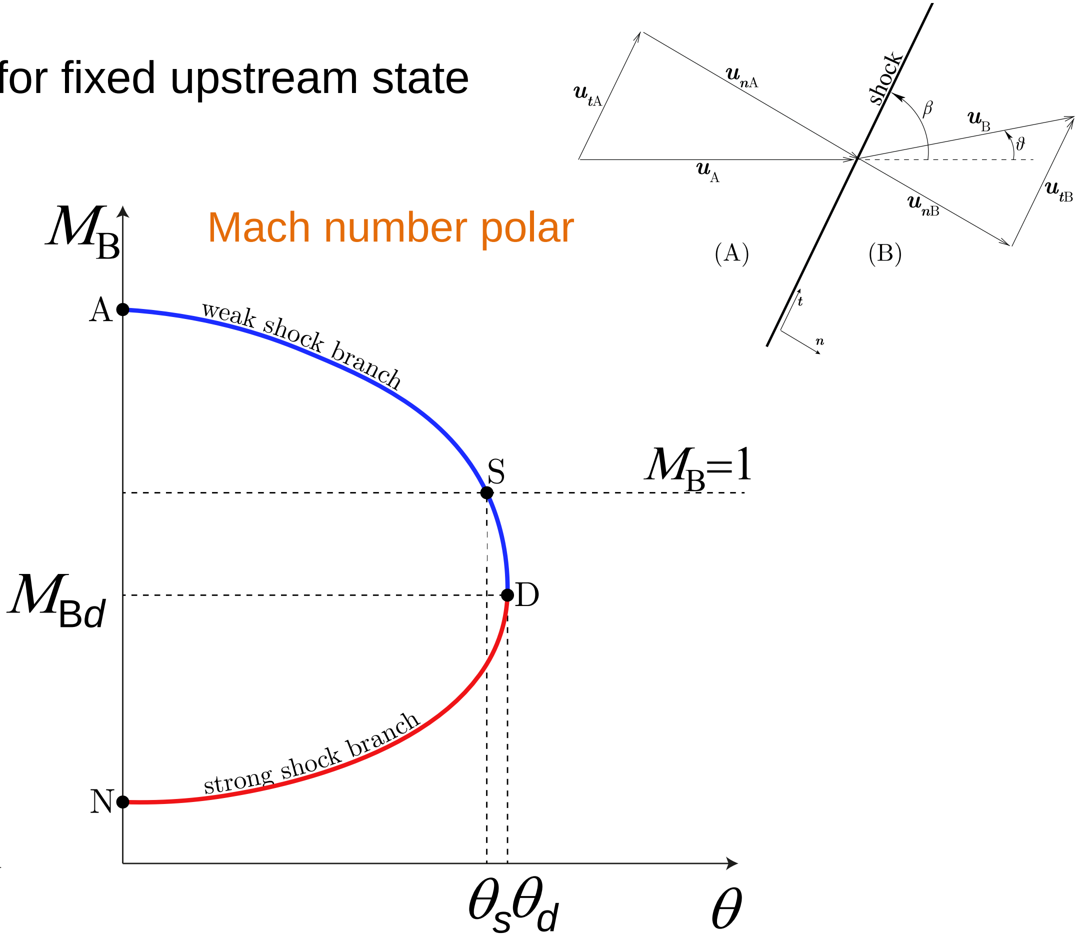
$$s_{\mathrm{B}} > s_{\mathrm{A}}$$

$$\left(M_{n\mathrm{B}} = \frac{|\boldsymbol{u}_{n\mathrm{B}}|}{c_{\mathrm{B}}} < 1 < \frac{|\boldsymbol{u}_{n\mathrm{A}}|}{c_{\mathrm{A}}} = M_{n\mathrm{A}} \right)$$

METHODOLOGY

Deflection shock polars: X-θ diagrams for fixed upstream state





A – Mach wave (acoustic limit)

N – normal shock

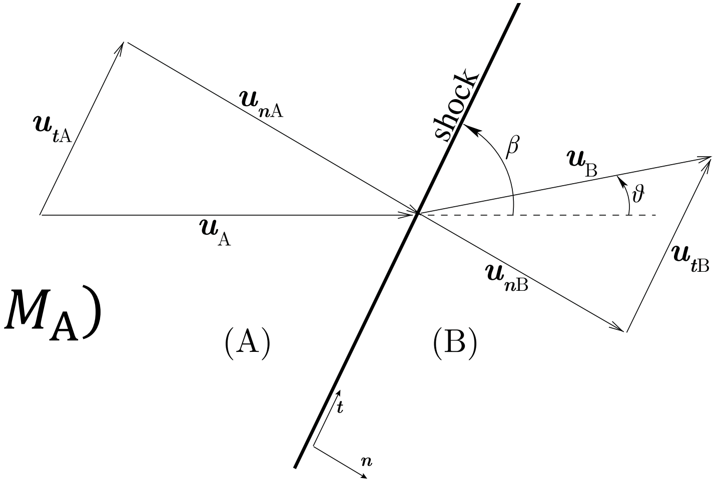
D – detachment point (max deflection)

S – downstream sonic point

OBLIQUE SHOCKS IN THE NON-IDEAL REGIME

Perfect-gas: explicit formulas

$$\tan\theta = \frac{2}{\tan\beta}\left[\frac{M_\mathrm{A}^2\sin^2\beta - 1}{M_\mathrm{A}^2(\gamma + \cos 2\beta) + 2}\right] \longrightarrow \beta(\vartheta, M_\mathrm{A})$$



$$\frac{\rho_\mathrm{B}}{\rho_\mathrm{A}} = \frac{(\gamma+1)M_\mathrm{A}^2\sin^2\beta}{2 + (\gamma-1)M_\mathrm{A}^2\sin^2\beta}$$

$$\frac{P_\mathrm{B}}{P_\mathrm{A}} = 1 + \frac{2\gamma}{\gamma+1}\left(M_\mathrm{A}^2\sin^2\beta - 1\right)$$

$$M_\mathrm{B}^2 = \frac{1}{\sin^2(\beta-\vartheta)}\,\frac{1 + \frac{\gamma-1}{2}M_\mathrm{A}^2\sin^2\beta}{\gamma M_\mathrm{A}^2\sin^2\beta - \frac{\gamma-1}{2}}$$

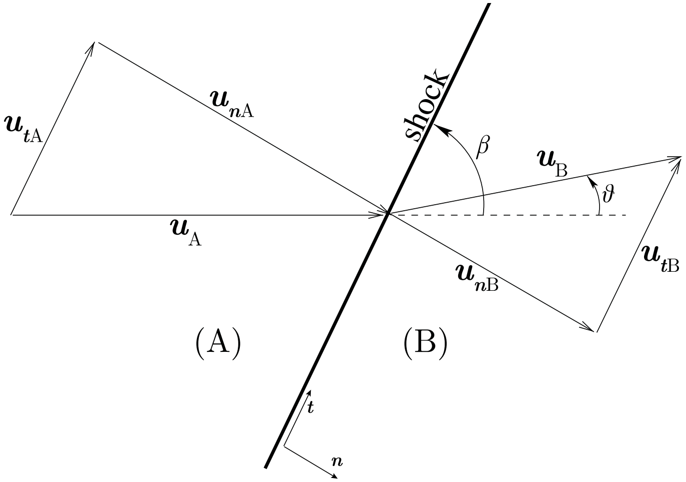
Dependencies:

- Deflection angle ϑ
- Upstream Mach number M_A

No dependence on the upstream thermodynamic state (e.g. $P_\mathrm{A}, \rho_\mathrm{A}$)

OBLIQUE SHOCKS IN THE NON-IDEAL REGIME

Non-ideal regime: acoustic limit



$$\beta = \sin^{-1}(1/M_{\mathrm{A}}) + \frac{\Gamma_{\mathrm{A}}}{2}\frac{M_{\mathrm{A}}^2}{M_{\mathrm{A}}^2 - 1}\vartheta + \mathcal{O}(\vartheta^2)$$

$$\frac{\rho_{\mathrm{B}}}{\rho_{\mathrm{A}}} = 1 + \frac{M_{\mathrm{A}}\Gamma_{\mathrm{A}}}{\sqrt{M_{\mathrm{A}}^2 - 1}}\vartheta + \mathcal{O}(\vartheta^2)$$

$$\frac{P_{\mathrm{B}}}{P_{\mathrm{A}}} = 1 + \frac{\rho_{\mathrm{A}} c_{\mathrm{A}}^2}{P_{\mathrm{A}}}\frac{M_{\mathrm{A}}\Gamma_{\mathrm{A}}}{\sqrt{M_{\mathrm{A}}^2 - 1}}\vartheta + \mathcal{O}(\vartheta^2)$$

$$M_{\mathrm{B}} = M_{\mathrm{A}} + \left(1 - \Gamma_{\mathrm{A}} - \frac{1}{M_{\mathrm{A}}^2}\right)\frac{M_{\mathrm{A}}^3}{\sqrt{M_{\mathrm{A}}^2 - 1}}\vartheta + \mathcal{O}(\vartheta^2)$$

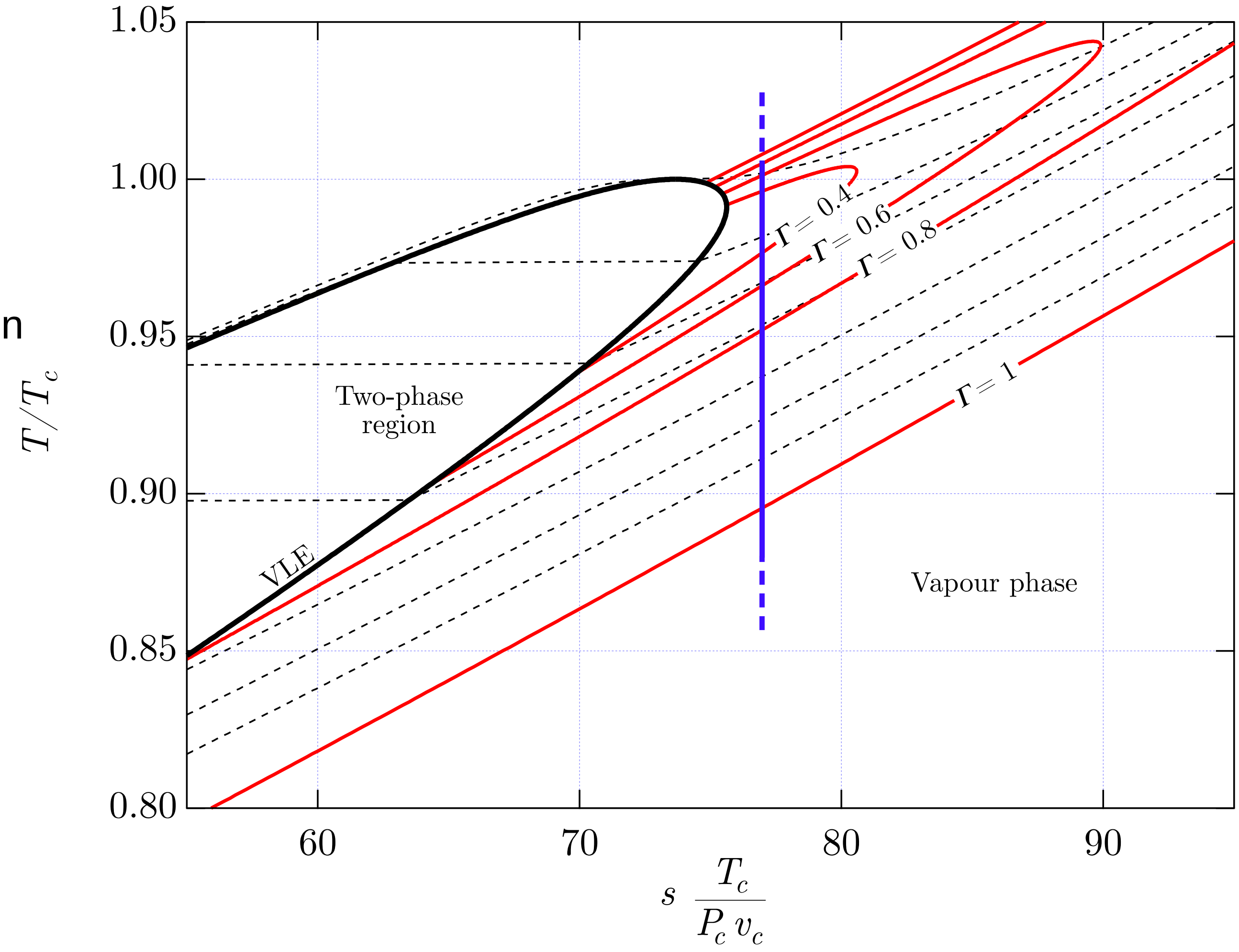
Dependences:

- Deflection angle ϑ
- Upstream Mach number M_{A}
- Upstream thermodynamic state (e.g., P_{A}, ρ_{A})

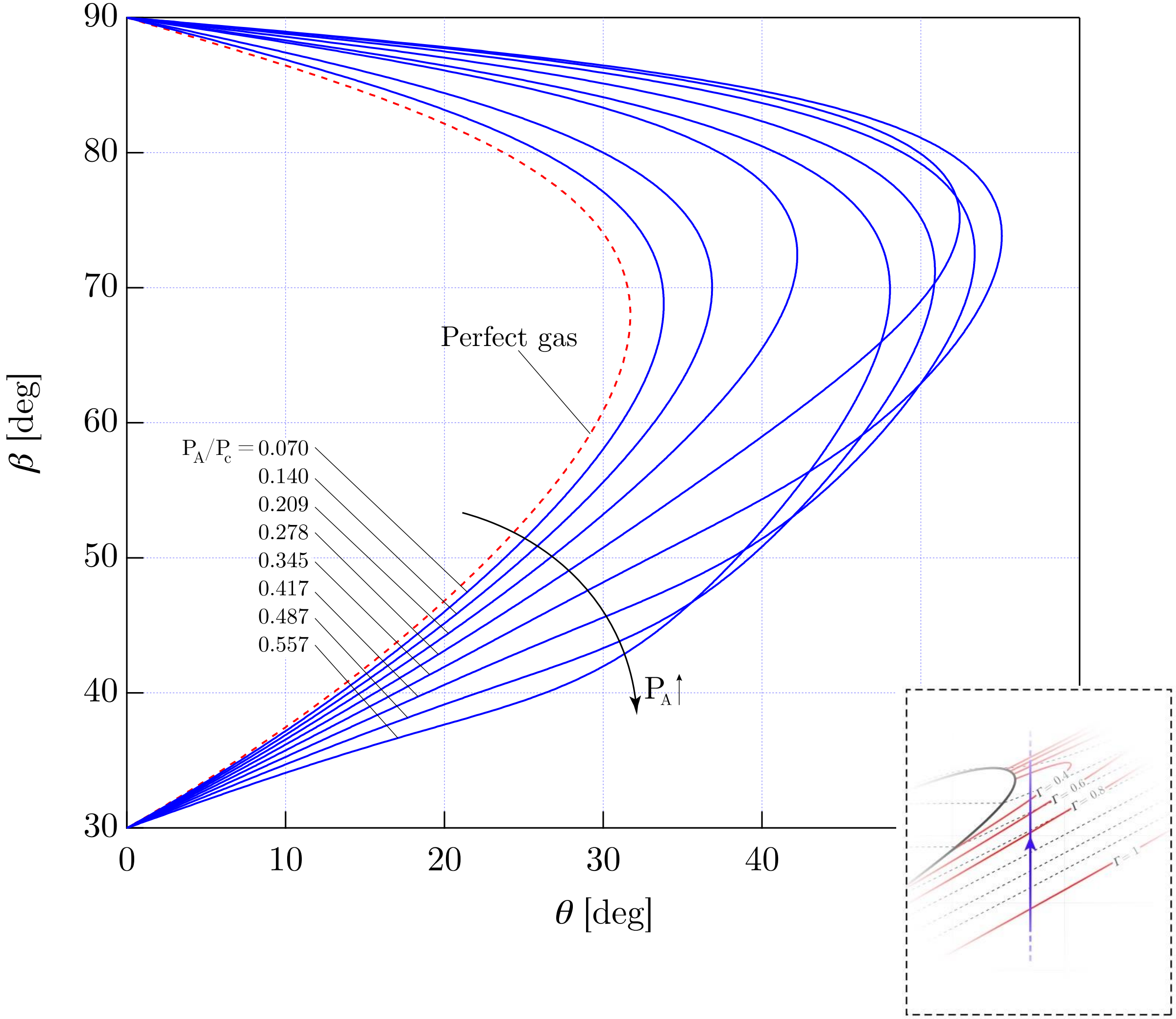
APPLICATION: OBLIQUE SHOCKS IN MDM

Parametric study

- Fluid: siloxane MDM (RefProp)
- Fixed upstream entropy in the non-ideal thermodynamic regime ($\Gamma < 1$)
- Fixed upstream Mach number ($M_A = 2$)



APPLICATION: OBLIQUE SHOCKS IN MDM



$\beta - \vartheta$ Diagram

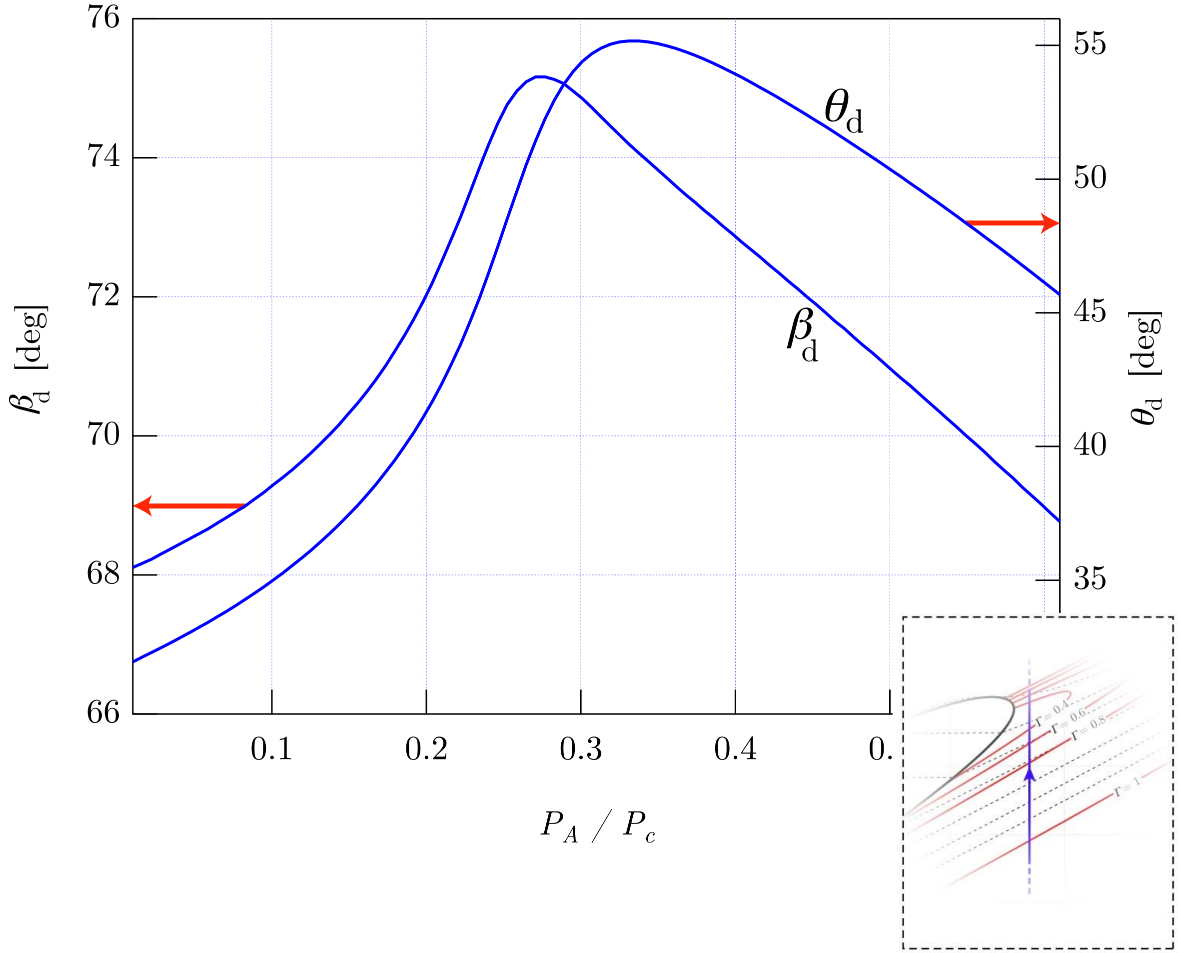
- Acoustic limit:

$$\beta = \sin^{-1}(1/M_{\mathrm{A}}) + \frac{\Gamma_{\mathrm{A}}}{2}\frac{M_{\mathrm{A}}^2}{M_{\mathrm{A}}^2 - 1}\vartheta$$

dependence on the upstream tmd state through Γ_{A}

- Strong dependence of the detachment angles on the upstream tmd state

APPLICATION: OBLIQUE SHOCKS IN MDM



$\beta - \vartheta$ Diagram

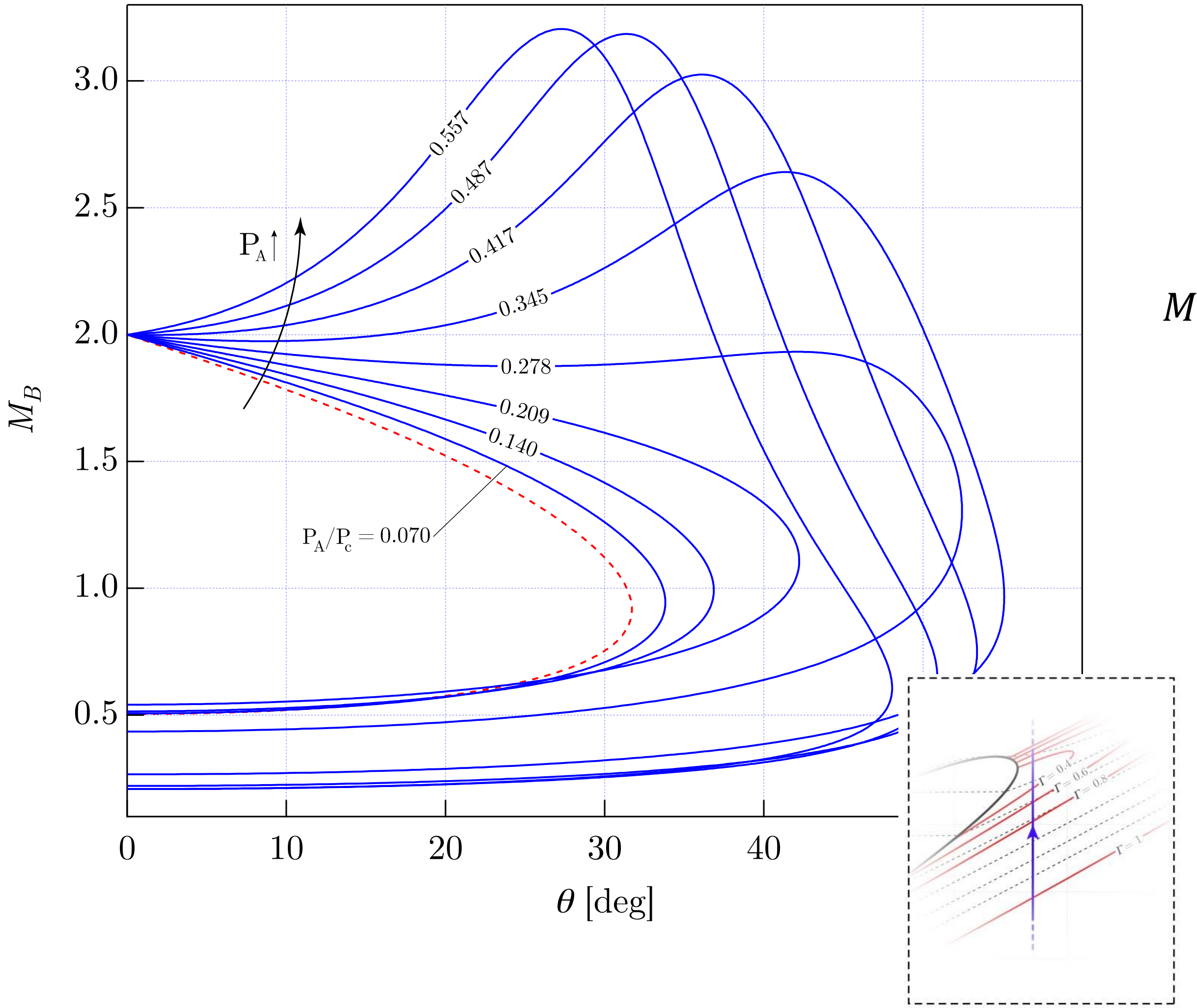
- ○ Acoustic limit:

$$\beta = \sin^{-1}(1/M_\mathrm{A}) + \frac{\Gamma_\mathrm{A}}{2}\frac{M_\mathrm{A}^2}{M_\mathrm{A}^2 - 1}\vartheta$$

dependence on the upstream tmd state through Γ_A

- ○ Strong dependence of the detachment angles on the upstream tmd state

APPLICATION: OBLIQUE SHOCKS IN MDM



$M_B - \vartheta$ Diagram

- ○ Acoustic limit:

$$M_B = M_A + \left(1 - \Gamma_A - \frac{1}{M_A^2}\right)\frac{M_A^3}{\sqrt{M_A^2 - 1}}\vartheta$$

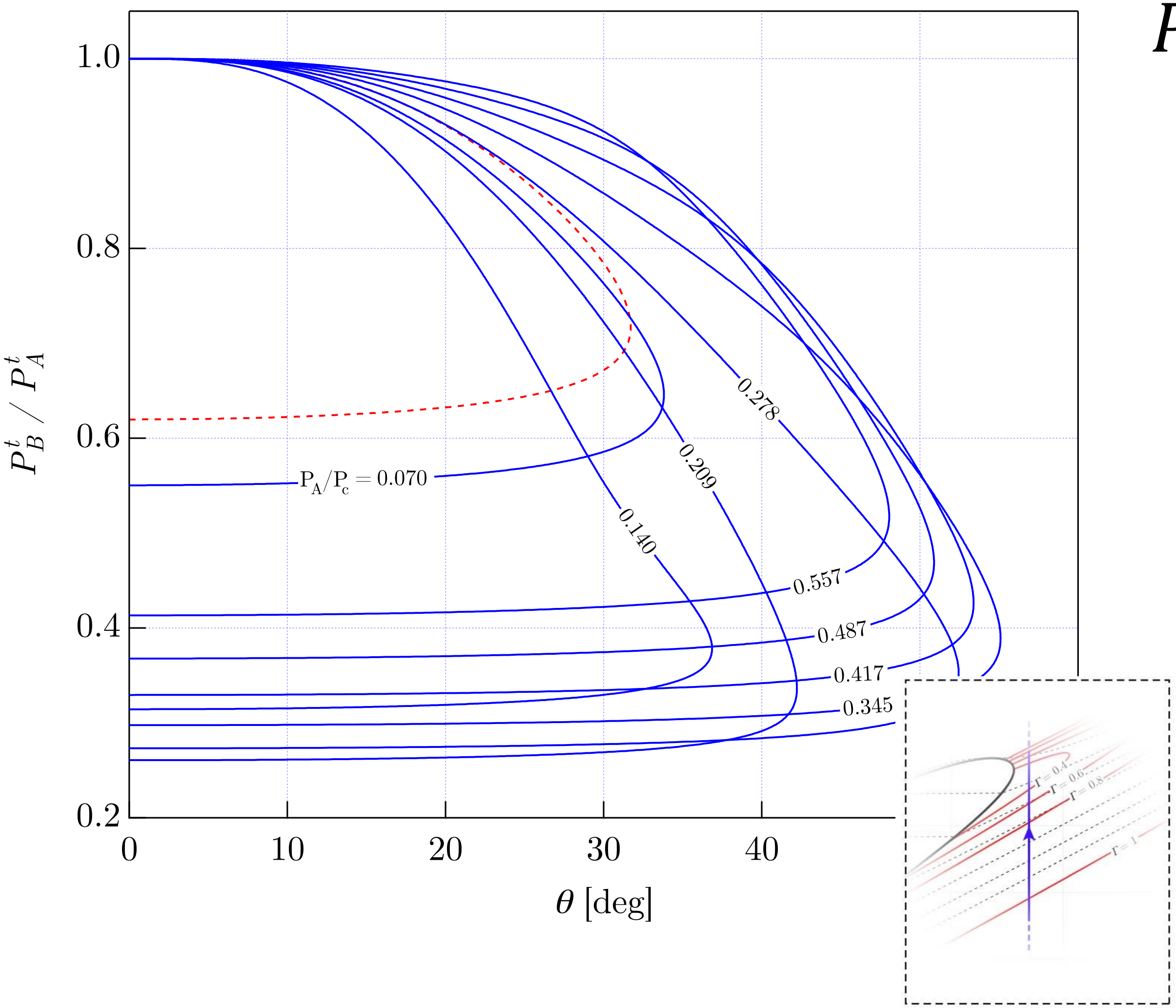
dependence on the upstream tmd state through Γ_A

- ○ Non-ideal oblique shocks (Mach number increasing):

$$\Gamma_A < 1 - \frac{1}{M_A^2}$$

for small deviations

APPLICATION: OBLIQUE SHOCKS IN MDM



$P^t_B/P^t_A - \vartheta$ Diagram

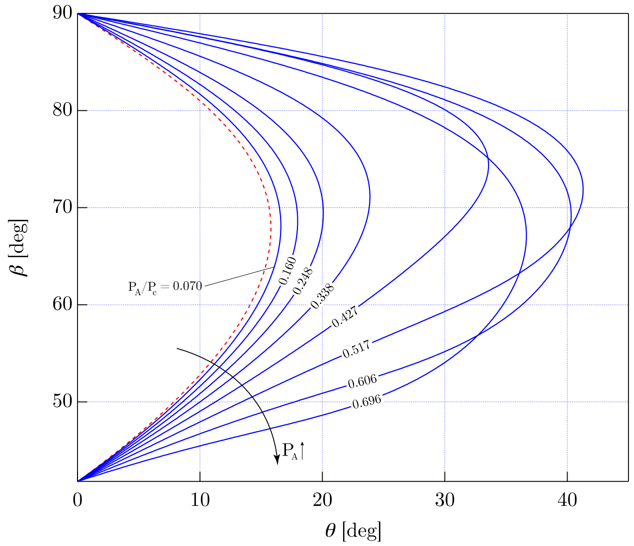
- Fixed ϑ → non-monotonic variation of the shock loss with the upstream pressure
- Larger shock loss across strong oblique shocks w.r.t perfect-gas case
- Smaller shock loss across weak oblique shocks w.r.t perfect-gas case for low values of Γ_A

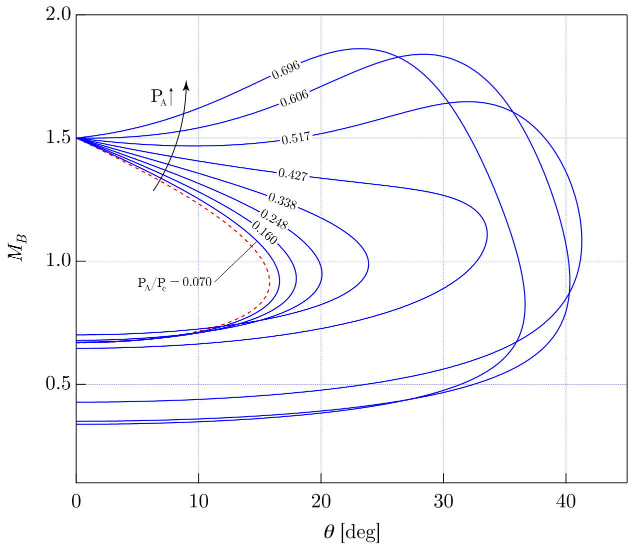
APPLICATION: OBLIQUE SHOCKS IN MDM

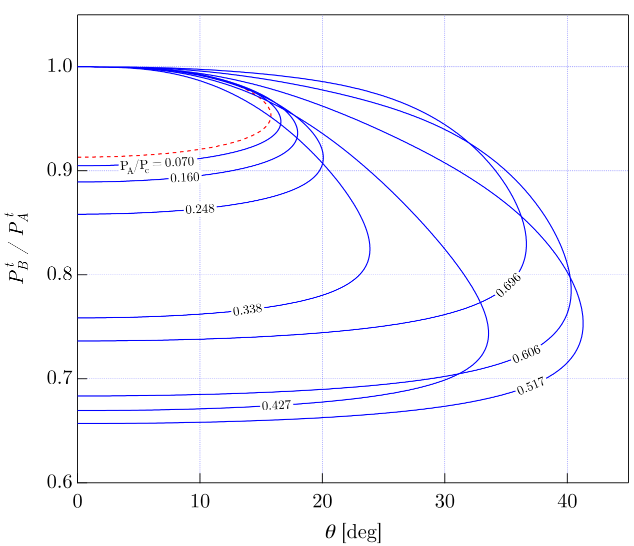
Further parametric study

- Same fluid, same isentrope, $M_\mathrm{A} = 1.5$
- Non-ideal oblique shocks (Mach number increasing): lower threshold on the upstream Mach number

$$M_{\mathrm{A,min}} = 1/\sqrt{1 - \Gamma_{\mathrm{A,min}}}$$





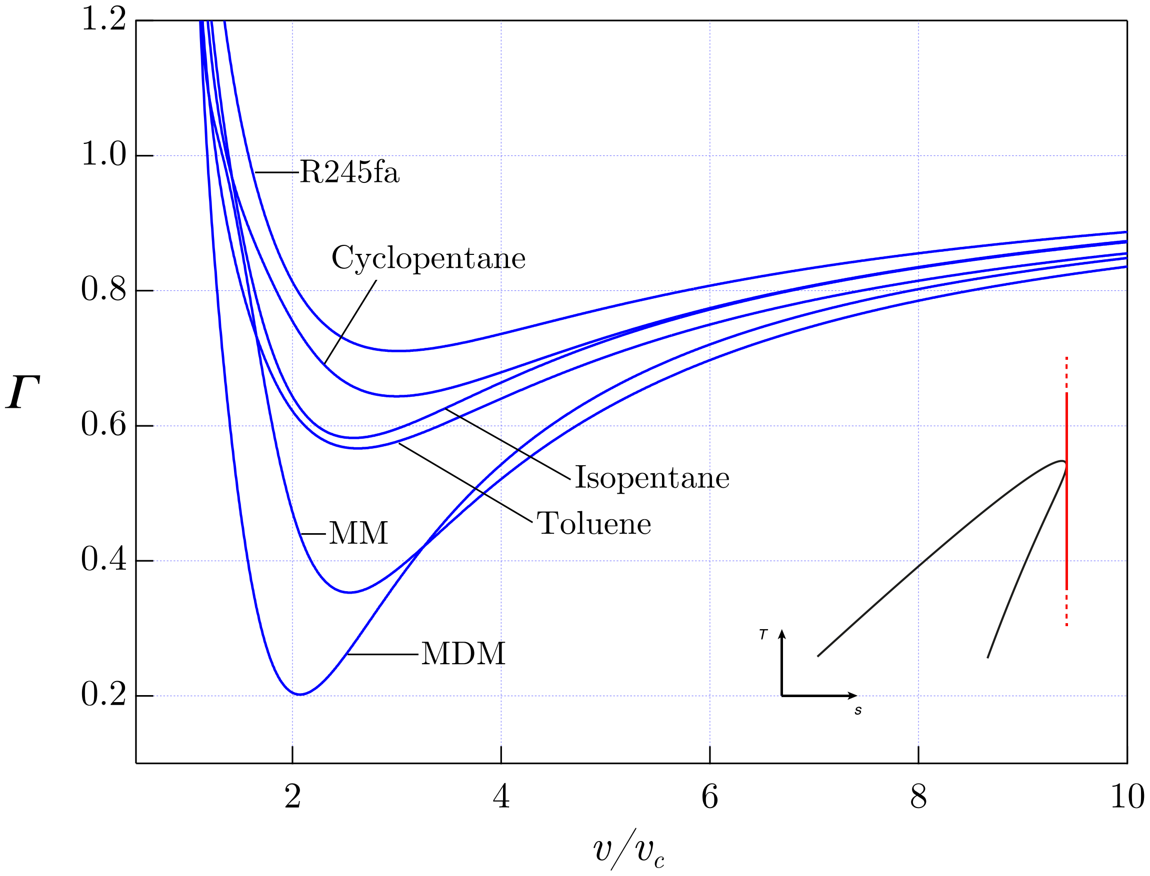


COMMENTS

Extension to other fluids

- Same qualitative behaviour expected for most moderate-to-high molecularly complex fluids

- Qualitatively similar thermodynamic topology of the fundamental derivative of gasdynamics



Extension to other fluids

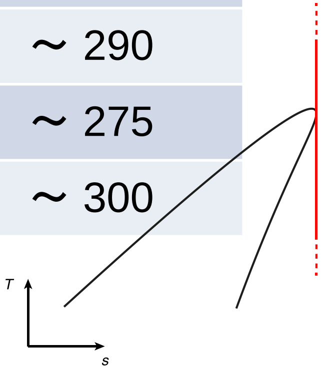
- Non-ideal oblique shocks (Mach number increasing): lower threshold on the upstream Mach number

$$M_{\mathrm{A,min}} = 1/\sqrt{1-\Gamma_{\mathrm{A,min}}}$$

- Total conditions for non-ideal oblique shocks may exceed thermal stability limit

Example: minimum P^t and T^t for non-ideal oblique shocks along isentrope tangent to VLE

Fluid	P^t_{min}[bar]	T^t_{min}[°C]	T_{lim}[°C]
MDM	16.55	299.0	∼ 290÷300
MM	25.90	262.0	∼ 300
Toluene	74.00	355.0	∼ 400
Isopentane	64.80	221.0	∼ 290
Cyclopentane	97.30	280.7	∼ 275
R245fa	107.77	204.0	∼ 300



CONCLUSIONS

- Oblique shock waves were investigated in the non-ideal gime because of their relevance to ORC turbine flows

- Main results:
  - Shock angle polar shifts to higher deflection angle
  - Appearence of Mach number-increasing oblique shocks (non-ideal oblique shocks)
  - Shock loss: larger across strong oblique shocks, possibly smaller across weak oblique shocks w.r.t. perfect-gas case

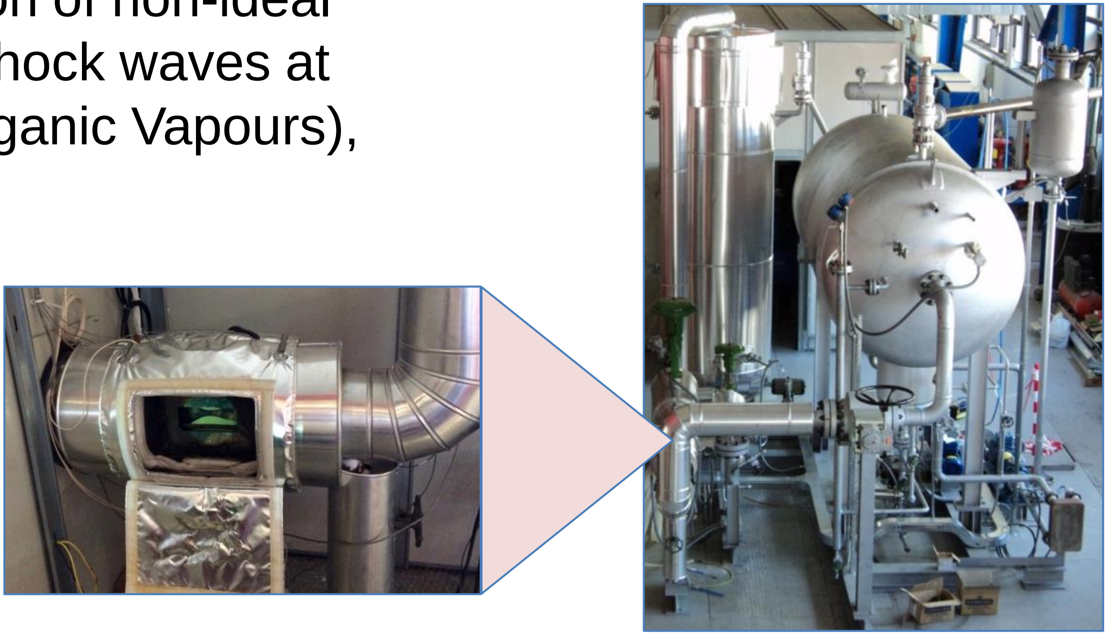
- MDM used for explanatory purposes, direct extension to other molecularly complex fluids employed in ORCs

- Highly non-ideal effects at design conditions only for supercritical ORCs

FUTURE WORK

- Numerical investigation on real vanes configurations at design and off-design conditions

- Experimental observation of non-ideal effects across oblique shock waves at TROVA (Test Rig for Organic Vapours), CREALab PoliMi



...QUESTIONS?

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